DIALECTICS AND TEMPORALITY IN MARX'S MATHEMATICAL MANUSCRIPTS.[1]

There have been many disputes in the history of value theory. Since the 1980s, a controversy has flared up between the Marxist supporters of an equilibrium approach SS which stresses that the capitalist economy either is in a state of, or tends towards, equilibrium SS and those Marxists who argue that the concept of equilibrium is theoretically alien to Marx's theory. For these latter authors, not only equilibrium but also the deviations from it (disequilibrium) are only powerful ideological notions without any relevance for an economic theory of the real world. The capitalist economy does not tend towards equilibrium but towards crises through the succession of economic cycles. These two views are radically different. The terms "(dis)equilibrium" and "non-equilibrium" economics underline the difference. The dispute has not been settled yet, either way.

The debate has focused mainly on two aspects: the so-called transformation problem and the tendential fall of the profit rate. In both cases, from the perspective of equilibrium and concomitant simultaneism (a theorization

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1 This paper is a modified version of Carchedi, forthcoming *b*, section 5. It has benefited from comments by Hans van den Bergh, professor of mathematics at the University of Wageningen. The usual caveat applies.

of the economy as if time did not exist, i.e., where everything happens simultaneously) many inconsistencies can be found in Marx. But from the perspective of non-equilibrium and concomitant temporalism (a theorization of the economy in which time in its irreversibility plays an indispensable role) these inconsistencies disappear.[2] The aim of this paper is not to revisit the debate. Suffice it to mention that temporalism recommends itself for the simple fact that it makes possible the solution of those inconsistencies that are introduced in Marx's analysis if time is cancelled, i.e., if one is interested in a theory of the real ("timefull") world rather than in a theory of a timeless (virtual) world. Neither is this article meant to address a different but related question: if the non-equilibrium and temporal approach is chosen, if the economy and thus society are neither in a state of, nor tend towards, equilibrium, how can the economy and thus society reproduce themselves? This question has been dealt with in a different work (Carchedi, forthcoming, a and b). In that work, a conception of dialectics as a method of social research is submitted

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2 For the transformation debate, see Ernst, 1982, Carchedi, 1984 and Freeman and Carchedi, 1996, where the relevant bibliography can be found. For a review of the debate up to the present and for an updated bibliography, see Kliman, 2007. For the rate of profit debate see Alberro and Persky, 1981; Cullenberg, 1994; Fine and Harris, 1976; Foley, 1986, 1999, 2000; Freeman, 1999; Kliman, 1996, 1999, 2007; Kliman and Freeman, 2000; Laibman, 1982, 1999a, 1999b, 2000a, 2000b; Reuten, 2004; Shaikh, 1978.

that accounts for both the reproduction and the possibility of supersession of capitalist society in the absence of the notion of (dis)equilibrium. Rather, this paper participates in the debate in an indirect way, by addressing a different question: can support for Marx's notion of dialectics be found in his Mathematical Manuscripts?

Marx never set out explicitly what his notion of dialectics was. Elsewhere I have argued that, rather then seeking it in Hegel, it should be extracted from Marx's own work. The conclusion reached is that that conception is based on three principles SS the coordinates of Marx's method of social research, as it were. First, all phenomena are always both realized and potential, i.e., they contain within themselves a realm of possibilities whose realization explains their change. The realized and the potential aspects can be contradictory. Second, they are always both determinant and determined. They are tied by a relation of mutual determination. The determinant phenomena are the condition of existence of the determined ones and these latter phenomena are the condition of reproduction or of supersession of the determinant ones. Third, phenomena are always subject to movement and change: they change from a realized to a potential state and vice versa and from a determinant to a determined state and vice versa. Movement and change imply, necessarily, time.

It follows that social reality, seen from the perspective of dialectical relations, is a temporal flow of determining and determined contradictory phenomena continuously emerging from a potential state to become realized and going back to a potential state. Society, and thus phenomena as its constituent elements, reproduce or supersede themselves through this movement powered by its internal contradictions. Neither equilibrium nor disequilibrium plays a role in society's reproduction. They are simply ideological constructions void of any scientific content. The dialectical method of social research, then, inquires into the origin, present state and further development of social phenomena, within this perspective of social reality.

On the basis of these preliminary concepts we can now deal with the question of this paper: do the Manuscripts support the dialectical method used by Marx as just set out?

Commentators generally focus on the Mathematical Manuscripts in order to inquire into Marx's own method of differential calculus from the perspective of the history of mathematics.[3] One of the questions raised by the commentators is why Marx embarked on such a study. As is well known, Marx explicitly located his interest in calculus in his perception that his knowledge was insufficient for his elaborations of the principles of economics.

3 See Alcouffe, 1985, 2001; Antonova, 2006; Blunden, 1984; Engels, 1983, 1987, 1990; Gerdes, 1983, Yanovskaya, 1969, 1983; Kennedy, 1977; Lombardo Radice, 1972; Smolinski, 1973.

Alcouffe (1985) holds that Marx liked mathematics as such because of its "rigor and intellectual gymnastics" (41) and that the recreational, playful and philosophical aspects of mathematics were for him at least as important as his preoccupation with economics (40). On the other hand, Yanovskaya, the most important commentator on the Manuscripts, remarks that the Manuscripts offer no answer as to what led Marx to move from the pursuit of algebra and commercial arithmetic to that of differential calculus (1969, 23). Marx was probably moved by more than one interest so that Alcouffe's thesis does not necessarily exclude Marx's explicitly stated reason. But there might be yet another reason, a more philosophical one. As it will be seen below, Marx's critique of differential calculus and the development of his own method of differentiation focus on the ontological nature of the infinitesimal. The thesis of this paper is that Marx, in studying differential calculus, was seeking support for, and material for the further development of, his method of social analysis. Seen from this angle, the Manuscripts are vastly more significant for the social scientist than for the mathematician or for the historian of mathematics.

The first evidence of Marx's interest in mathematics is contained in a letter to Engels of 1858 in which he wrote: "In working out economic principles I have been so damned delayed by mistakes in computation that out of despair I have begun again a quick review of algebra. Arithmetic was always foreign to me. By the algebraic detour I am shooting rapidly ahead again." In 1863 he wrote, again to Engels: "In my free time I do differential and integral calculus." Most interestingly, in another letter to Engels 10 years later (1873), he provides an example of what economic principles he had in mind:

> I have been telling Moore about a problem with which I have been racking my brains for some time now. However, he thinks it is insoluble, at least pro tempore, because of the many factors involved, factors which for the most part have yet to be discovered. The problem is this: you know about those graphs in which the movements of prices, discount rates, etc. etc., over the year, etc., are shown in rising and falling zigzags. I have variously attempted to analyze crises by calculating these ups and downs as irregular curves and I believed (and still believe it would be possible if the material were sufficiently studied) that I might be able to determine mathematically the principal laws governing crises. As I said, Moore thinks it cannot be done at present and I have resolved to give it up for the time being.

In light of the fact that "the principal laws governing crises" are, as all social laws, tendential and contradictory, "to determine mathematically" the laws is an impossible task. First, mathematics is a branch of formal logic and premises in formal logic cannot be contradictory. However, to account for the laws of movement in society one has to start from contradictory premises (in the sense of dialectical contradictions) and this is why the laws of movement are tendential. Second, even if all the "factors involved" were known, it would be practically impossible to consider all of them. This is why econometric models, even large ones involving thousand of relations, have such a dismal record as tools of prediction. But if it is impossible to determine the laws of crises purely in terms of mathematics, it is certainly possible to analyze the cyclical movement of economic indicators (the ups and downs) by using "higher mathematics." This was Marx's intuition.

At this juncture, two further questions arise. First, why did Marx make no use of differential calculus in his work? According to Smolinski, for Marx

> the key fact is that a commodity has value or does not have it, labor is productive or is not, a participant in the economic process is a capitalist or a proletarian, society is capitalist or socialist. For this polarized universe a binary calculus might be a more suitable tool than differential calculus. (1973, 1199.)

However, Alcouffe remarks that the reproduction schemes and the tendential fall of the profit rate are amenable to be treated with the mathematics developed by Marx. For example, differential calculus can be used to compute the instantaneous rate of change in the profit rate (1985, 37). Both opinions seem to have an element of truth. Differential calculus is indeed applicable to some aspects of Marx's economic theory but the question is whether this would be relevant. The relevant question is not how the rate of profit changes instantaneously but how it changes due to the dialectical interplay between the tendency and the counter-tendencies.[4] A more probable explanation is that, given that Marx finally mastered calculus towards the end of his life, he did not have the time and opportunity to write an analysis of the quantitative aspects of economic life (for example, of the economic cycle, the "zigzags" as he puts it in the letter cited above).

The second question is how Marx would have applied calculus had he had the time and opportunity to do so. This question cannot be settled by considering how mathematics has been applied in economic planning by formally centrally planned economies. As Smolinski reports, "According to a widely held view, it was Marx's influence that has delayed by decades the development of mathematical economics in the economic systems of the Soviet type, which, in turn, is said to adversely affect the efficiency with which they operate" (1973, 1189). But, as the author rightly points out and as the Manuscripts show, Marx was far from being ignorant of calculus and was greatly interested in its application to economics. It is true that

> the planners' "mathematicophobia," to use L. Kantorovich's apt expression, led to a substantial misallocation of resources

4 This point differs from Alcouffe's opinion that a formal mathematical treatment of the law of the tendential fall in the profit rate would be "particularly welcome" (1985, 37).

> through nonoptimal decisions. . . . The intellectual cost of the taboo in question was also high: reduced to a status of a "qualitative," dequantified science, economics stagnated. . . . [Oskar Lange] pointed out that Soviet economics degenerated into a sterile dogma, the purpose of which became "to plead the ruling bureaucracy's special interests and to distort and falsify economic reality." These processes led to "a withering away of Marxism. . . . Marxist [economic] science was replaced by a dogmatic apologetics." (Ibid.)

There is considerable confusion here. While Marx cannot be held responsible for the insufficient application of mathematics in Soviet-type economies and while this insufficiency was certainly an obstacle to the efficient functioning of an economic system, the reasons for the demise of the USSR and other Soviet-type centrally planned economies should be sought elsewhere. In short, in spite of its specific features including the absence of the market, the USSR had become a system where the politicalSmanagerial class was performing the function of capital. The application of planning techniques was meant to mirror the market as an allocation system. It was thus opposite to a system based on the laborers' self-management of the economy and society. Contrary to Smolinski's view, the planners' choices were often mistaken not because they "reflected the mistaken labor theory of value" (op. cit., 1190), but because an inherently capitalist system needed the market as an allocation system rather than any other type of allocation system. The optimal allocation for capital can only be through the market. The system was thus inherently weak and unable to compete with fully developed capitalist systems (Carchedi, 1987).[5]

As for Marx, the important question here is not whether and how Marx

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5 "Study of Marx's Mathematical Manuscripts had a major impact on Soviet research in the history and philosophy of mathematics, beginning in the 1930s. This was especially true in philosophy of mathematics, where virtually all of the work published between 1930 and 1950 dealt with the manuscripts. The history of mathematics, however, also received considerable stimulation due to what Marx had written. . . . Thus the significance of the discovery and study of the mathematical papers of Karl Marx in the Soviet Union may be assessed in several different ways. To the extent that editorial work on the manuscripts promoted study in the 1930s of the history of mathematics, its effect was positive. In particular, the manuscripts provided a strong rationale for serious examination of the history of analysis. It also followed that to appreciate Marx fully, it was necessary to study the history of mathematics in general. Unfortunately, where foundations of mathematics are concerned, Marx and the manuscripts have had a largely negative impact. This has been due primarily to the tendency of foundational research to focus almost exclusively on dialectical interpretations of mathematics according to Marx's fundamental doctrines. As for the technical, internal development of mathematics itself, Marx's manuscripts do not seem to have played any appreciable role, positive or negative" (Dauben, 2003, 2S3).

would have applied differential calculus to his economic theory. This is scarcely important. Rather, the point is that even though the Manuscripts do not deal with the relation between dialectics and differential calculus, Marx's method of differentiation provides key insights into what was Marx's dialectical view of reality. This point has escaped all the commentators on the Manuscripts. Yet, it is these insights rather than Marx's own original method of differentiation that are the really important aspect of the Manuscripts.

Let us begin by considering how "Leibniz arrived at the notion of derivative . . . from geometric considerations" (Gerdes, 1985, 24; Struik, 1948, 187ff.). Let $y_1 = x_1^3$. Starting from $\mathrm{d}x = x_1 - x_0$ and $\mathrm{d}y = y_1 - y_0$,

$$y_1 = x_1^3 = (x_0 + \mathrm{d}x)^3 = x_0^3 + 3x_0^2\mathrm{d}x + 3x_0(\mathrm{d}x)^2 + (\mathrm{d}x)^3. \qquad (1)$$

Given that $y_0 = x_0^3$, we have

$$y_1 = y_0 + 3x_0^2\mathrm{d}x + 3x_0(\mathrm{d}x)^2 + (\mathrm{d}x)^3, \qquad (2)$$

so that

$$y_1 - y_0 = \mathrm{d}y = 3x_0^2\mathrm{d}x + 3x_0(\mathrm{d}x)^2 + (\mathrm{d}x)^3, \qquad (3)$$

and dividing both sides by dx, we obtain:

$$dy/dx = 3x_0^2 + 3x_0dx + (dx)^2. \quad (4)$$

At this point, following Leibniz, we can eliminate terms containing dx on the right, given that dx is infinitely small. Thus, we obtain, for the derivative,

$$dy/dx = 3x_0^2\text{, or more generally, } 3x^2 \quad (5)$$

(Gerdes, 1985, 24S30). The problem, according to Marx, is twofold. First, the derivative $3x_0^2$ already appears in equation (1), i.e., before the derivation, before dx is set equal to zero. Thus, to get the derivative, "the terms which are obtained in addition to the first derivative [$3x_0dx+(dx)^2$] . . . must be juggled away to obtain the correct result [$3x_0^2$]" (Marx, 1983, 91). This is necessary "not only to obtain the true result but any result at all" (93). Marx calls this the "mystical" method. Second, if dx is an infinitesimally small quantity, if it is not an ordinary (Archimedean) number, how can we justify the use of the rules for ordinary numbers, e.g. application of the binomial expansion to $(x_0+dx)^3$? More generally, what is the theoretical and ontological status of infinitesimally small quantities?

In dealing with these difficulties, Marx develops his own method of derivation. Basically, Marx's method is as follows. Given a certain function, such as $y=f(x)$, Marx first lets x_o become x_1. Both x and y increase by finite quantities, Δx and Δy (so that the rules for ordinary numbers can be applied here). The ratio $\Delta y/\Delta x = [f(x_1)\ \mathrm{S}\ f(x_0)]/(x_1\ \mathrm{S}\ x_0)$ is what he calls the provisional or preliminary derivative. Then, he lets x_1 return to x_0 so that $x_1\ \mathrm{S}\ x_0 = 0$ and thus $y_1\ \mathrm{S}\ y_0 = 0$, thus reducing this limit value to its absolute minimum quantity. This is called the definitive derivative, dy/dx (so that the derivative appears only after the process of differentiation).[6] "The quantity x_1, although originally obtained from the variation of x, does not disappear; it is only reduced to its minimum limit value $= x$" (op. cit., 7, emphasis added). Let us then see how Marx computes the derivative of $y = x^3$.

If x_0 increases to x_1, y_0 increases to y_1. Given that $x_1\ \mathrm{S}\ x_0 = \Delta x$ and $y_1\ \mathrm{S}\ y_0 = \Delta y$,

$$\Delta y/\Delta x = (y_1\ \mathrm{S}\ y_0)/(x_1\ \mathrm{S}\ x_0) = (x_1^3\ \mathrm{S}\ x_0^3)/(x_1\ \mathrm{S}\ x_0). \qquad (6)$$

Now since

$$(x_1^3\ \mathrm{S}\ x_0^3) = (x_1\ \mathrm{S}\ x_0)(x_1^2 + x_1x_0 + x_0^2), \qquad (7)$$

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6 For a mathematically more precise formulation of Marx's method, see Marx, 1983, note 7, 195S6.

we substitute (7) into (6) to obtain:

$$\Delta y/\Delta x = [(x_1 \text{ S } x_0)(x_1^2 + x_1x_0 + x_0^2)]/(x_1 \text{ S } x_0) \qquad (8)$$

and we get the provisional derivative

$$\Delta y/\Delta x = x_1^2 + x_1x_0 + x_0^2. \qquad (9)$$

To get the definitive derivative, x_1 goes back to x_0 so that $\Delta x = \mathrm{d}x = 0$ and $\Delta y = \mathrm{d}y = 0$. Equation (9) thus becomes

$$\mathrm{d}y/\mathrm{d}x = x_0^2 + x_0^2 + x_0^2 = 3x_0^2. \qquad (10)$$

The definitive derivative is thus the "preliminary derivative reduced to its absolute minimum quantity" (<u>ibid</u>.). The two methods lead to the same results, but there are differences between them. First, "the starting points . . . are the opposite poles as far as operating method goes" (Marx, 1973, 68). In one case it is $x_0 + \mathrm{d}x = x_1$ (the "positive form"); in the other (Marx) it is x_0 increasing to x_1, <u>i.e.</u>, $x_1 \text{ S } x_0 = \Delta x$ (the "negative form") (<u>op. cit.</u>, 88). "One expresses the same thing as the other: the first negatively as the difference Δx, the second positively as the increment *h*" (128). In the positive form, "from the beginning we interpret the difference as its opposite, as a sum” (102). Second, the procedures differ too: the fraction $\Delta y/\Delta x$ is transformed into dy/dx and the derivative is obtained after the derivation, after x_1 is reduced to its absolute minimum quantity. In the positive method (form) “the derivative is thus in no way obtained by differentiation but instead simply by the expansion of $f(x + h)$ or y_1 into a defined expression obtained by simple multiplication” (104).

It could be argued that these differences are insignificant, given that both use only elementary algebra and divide the increment of a quantity, y, that depends on another quantity, x, by the increment in x.[7] Moreover, from a mathematical viewpoint Marx’s method is of limited applicability, “because it is often impossible to divide $f(x_1)$ S $f(x_0)$ by x_1 S x_0” (Gerdes, 1985, 73). Yet, it could also be argued that Marx’s method is of historical significance. Marx’s procedure allows him to realize that dy/dx is not a ratio between two zero’s, but rather a symbol indicating the procedure of first increasing x_0 to x_1 (and thus y_0 to y_1) and then reducing x_1 (and thus y_1) to their minimum values, x_0 and y_0. Marx’s discovery that dy/dx is an operational symbol anticipated “an idea that came forward again only in the 20th century”

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7 I owe this point to Hans van den Berg in a private communication.

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(Kolmogorov, quoted in Gerdes, 1985, 75).[8] Marx's stress on d*y*/d*x* as being an operational symbol, the "expression of a process" (op. cit. 8), the "symbol of a real process" (9) is a real achievement, an outstanding critique of the "mystical" foundations of infinitesimal calculus, of the metaphysical nature of infinitely small entities which are neither finite not null (Lombardo Radice, quoted in Ponzio, 2005, 23).

Be that as it may, these considerations are only of marginal interest for our present purposes. The point is that the analysis of this method offers important insights into Marx's notion of dialectics, as summarily sketched above.[9] Let us then see how these principles emerge implicitly from the Manuscripts.

First, for Marx the notion of an infinitesimally small quantity, of an infinite approximation to zero, of something that is neither a number nor zero, should be rejected as "metaphysical," as a "chimera." In his method, x_0

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8 According to Lombardo Radice, Marx did not know the critical foundations of analysis, from Cauchy to Weierstrass, something which emphasizes his "geniality" in criticizing autonomously the "mystical" foundations of calculus (1972, 274).

9 A detailed treatment can be found in Carchedi, forthcoming a, b. This view differs from Alcouffe's opinion that "the formalization of a social, and in particular of a critical science" should be sought in Hegel's Science of Logic (1985, 104). As argued in Carchedi, forthcoming a, b, it should be sought in and extracted from Marx's own work.

is first increased to x_1 (i.e., by dx) and then x_1 is reduced to x_o so that x_1 does not disappear but is reduced to its minimum limit value, x_0. Thus, dx, rather than being at the same time zero and not zero, is first a real number and then is posited equal to zero. This is the theorization of a temporal, real process. In this way Marx escapes the "chimerical" notion of derivative. The notations dx = 0 and dy = 0 are the symbols of this process, not real numbers divided by zero.[10]

Second, in the "positive" form motion is the result of a (small) quantity (dx) added to x_0, which is a constant. Implicitly, x_0 remains constant throughout, so that movement and change affect only a limited section of reality.[11] The starting point is a constant, a lack of movement and change, to

10 A similar point is made by Yanovskaya: "some scientists explained the infinitesimals or infinitely small quantities in terms of the dialectical nature of opposites SS at the same time equal to zero and different from zero." Yanovskaya called these scientists "pseudo-Marxists because they forgot that dialectical materialism does not recognize static contradictions (= 0 and ≠0), but only contradictions connected with motion" (Gerdes, 1985, 115S6).

11 In a letter to Marx dated 1882, Engels writes: "the fundamental difference between your method and the old one is that you make x change into x', thus making them really vary, while the other way starts from $x + h$ which is always only the sum of two magnitudes, but never a variation of a magnitude."

which change is added only as an appendix. This is a view of a static reality only temporarily disturbed by a movement that moreover applies only to an infinitesimal part of reality. The analogy with equilibrium and disequilibrium (temporary deviations from equilibrium) in the social sciences is clear. d*x* is added to *x* from outside *x*. Movement is not powered by the internal nature and structure, but is the result of external forces. Behind the "positive form" lays a static interpretation of reality, behind the alternative a dynamic view.

For Marx "x_1 is the increased *x* itself; its growth is not separated from it. . . . This formula distinguishes the increased *x*, namely x_1, from its original form prior to the increase, from x_0, but it does not distinguish *x* from its own increment" (Marx, 1983, 86). In Marx's method, it is the whole, x_0, that moves, that grows to x_1 by d*x*. The movement from x_0 to x_1 (Marx's starting point) and back (the end point) indicates a change in the whole of reality, even if caused by a minimal part of it. x_0 cannot increases by Δx (or d*x*) without changing into x_1; the change in a part of reality (however small) changes the whole of it due to the interconnection of all of reality's constituent parts. This is a dynamic view in which absence of movement and change play no part. x_0 can grow to x_1 only because $x + dx$ is inherent in *x* as one of its potentialities. Marx's method, then, implies that *x* contains potentially within itself $x + dx$, that this latter realizes itself as $x + dx$, and that if $x + dx$ returns to *x* it becomes again a potential inherent in *x*. Even though not explicitly stated by

Marx, his method presupposes that aspect of dialectics submitted here that distinguishes between realized and potentials.[12] The fact that this might not be the way modern mathematics conceptualizes d*x* is irrelevant for this article.

To sum up. Which view of social reality is hidden behind and informs Marx's method of differentiation? Marx differentiates with the eyes of the social scientist, of the dialectician. His method of differentiation mirrors a process that is real, temporal, in which something (a real number) cannot be at the same time also something else (zero) and in which movement affects the whole rather than only a part of it and is the result of the interplay of potentials and realized. Marx's method of differential calculus is consonant only with a dynamic and temporal approach (and inconsistent with an approach in which time does not exist, as in simultaneism in economics) and more generally with the notion of dialectics summarily sketched above and developed in detail elsewhere (Carchedi, forthcoming a, b). This conclusion is highly relevant for the debate between those Marxists who hold that in Marx's theory time is the essential coordinate of a dynamic, non-equilibrium, system and those who adhere to a theory in which time and movement are absent.

12 In social reality, on the contrary, a social phenomenon can decrease in size until it becomes an individual phenomenon, a potential social phenomenon. See Carchedi, forthcoming a. But in social reality the notion of infinitesimally small is nonsensical.

The question is not whether Marx's method (in any case, correct within its limits) is relevant for mathematics or for the history of mathematics.[13] The question is that the manuscripts are highly relevant for the social scientists interested in uncovering and further developing Marx's own notion of dialectics as a method of social research and as a tool of social change.

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13 In a very interesting article, Dauben draws attention to the link between nonstandard analysis and Marx's mathematical manuscripts in China: "Nearly a century after Marx, Chinese mathematicians explicitly linked Marxist ideology and the foundations of mathematics through a new program interpreting calculus using infinitesimals, as Marx had advocated, but now in the rigorous terms of nonstandard analysis, the creation of Abraham Robinson in the 1960s. During the Cultural Revolution (1966S1976), mathematics was suspect in China for being too abstract, aloof from the concerns of the common man and the struggle to meet the basic needs of daily life in a still largely agrarian society. However, when Chinese mathematicians discovered the mathematical manuscripts of Karl Marx, these seemed to offer fresh grounds for justifying abstract mathematics, especially concern for foundations and critical evaluation of the calculus" (Dauben, 2003, 328). Notice that this would seem to provide no answer to what was essentially Marx's question, i.e., the ontological nature of infinitely small or large numbers. The hypothesis that there is a "cloud" of hyperreal numbers floating infinitesimally close to each number on the real line leaves Marx's question unanswered.

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