Logic and Dialectics in Social Science, Part II: Dialectics, Formal Logic and Mathematics[1]

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Abstract

Building upon the theory submitted in the previous issue of this journal, this article examines both similarities and differences with Engels' dialectics of nature. It argues that Engels' approach is unsuitable to reveal the specificity of society's movement towards both reproduction and supersession. It then considers the basic features of formal logic and compares them with dialectical logic. It stresses the class nature of formal logic and the conditions upon which the tools of formal logic (rather than formal logic itself) can be employed within dialectical reasoning. Particular attention is given to deduction and verification in dialectical and formal logic. Finally, Part II looks at Marx's *Mathematical Manuscripts* and argues, differently from other commentators, that the *Manuscripts'* real importance resides not in Marx's original method of differentiation but in their providing key insights into Marx's notion of dialectics. These insights are found to support the theory submitted in this work.

Keywords
dialectical logic, dialectics of nature, formal logic, Marx's mathematical manuscripts, verification

Dialectics of Nature

The first part of this article (Carchedi, 2008) has dealt with the notion of dialectics as a method of social research. A full appreciation of this method requires that similarities and differences be addressed both with Engels' notion of dialectics and with formal logic. These are the themes of this second part which will also consider whether Marx's mathematical manuscripts lend support to the dialectical view and method submitted in this work. Finally, a critique of homo economicus from a non-equilibrium stance will conclude this work. Let us begin with Engels' dialectics of nature.

For Engels, the 'most general laws of dialectics' can be reduced to three. First, 'the law of the transformation of quantity into quality and vice versa'. Second, 'the law of the

interpenetration of opposites'. Third, 'the law of the negation of the negation' (1987: 356). These laws are reflections in thought of reality and are therefore 'real laws of development of nature ... valid also for theoretical natural sciences' (1987: 357). There is some correspondence between the present approach and Engels' notion of dialectics. Engels' first law (the transformation of quantity into quality and vice versa) corresponds to the transformation of an aggregation of individual phenomena (potential social phenomena) into realized social phenomena and vice versa (see Part I, Section 2). Engels' second law (the interpenetration of opposites) corresponds to this approach's first principle, the relation between determinant and determined phenomena if the latter are conditions of supersession of the former (Part I, Section 1). And Engels' third law (the negation of the negation) corresponds to that aspect of mutual determination in which A calls into realized existence B as the condition of supersession of A (negation). Afterwards, B realizes the actual supersession of A (negation of the negation). In short, A negates itself in B and B negates A (Part I, Section 1). Another possible point of convergence is the notion of potential reality. Engels does not theorize potential reality explicitly, but it could be argued that that notion is inherent in his view. Water can become steam if boiled at 100 degrees centigrade. Or, 'The plant, the animal, every cell is at every moment of its life identical with itself and yet becoming distinct from itself ... by a sum of incessant molecular changes ... even in inorganic nature identity as such is in reality non-existent. Every body is continuously exposed to mechanical, physical and chemical influences, which are always changing it and modifying its identity' (1987: 495). Thus, it could be submitted that for Engels change is due to the reciprocal interaction of realized instances (e.g. molecules) but these changes are already potentially present before their realization. In terms of the present approach, at any given moment something is identical to itself (as a realized entity) and potentially different from itself.

But there are also fundamental differences. First, the three principles of dialectics on which this approach is based (Part I, Section 1) differ from Engels' dialectics because they are extracted from Marx's work and thus from a *class* analysis of social reality, rather than from the study of nature (in nature social classes do not exist). They apply only to the social sciences (and thus to the class analysis, i.e. the social determination, of natural sciences and of knowledge in general) rather than, as Engels holds, to both the natural and the social sciences (1987: 361).[2] Second, the notion of the tendential nature of society's laws of movement is lacking in Engels. Third, the notion of the determinant and determined aspects of phenomena is also lacking. And fourth, while there is no explicit denial of the social determination of natural sciences in Engels, natural sciences seem to develop according to their internal logic: progress in these sciences consists in overcoming errors and getting an increasingly correct insight in the laws of nature. This presupposes that the development of science, both social and natural, is not class determined (it is class neutral) whereas it can be argued that for Marx knowledge, all knowledge, is class determined.[3] This is a huge topic that can only be hinted at here. It concerns nothing less than the production of knowledge, a theoretical underdeveloped area in Marxist theory (and not only). All that can be said here is that the myth of the neutrality of science has led to the myth of the neutrality of the productive forces which, basically thanks to Engels, has

been accepted by the great majority of Marxists, including Lenin. It became possible, thus, for the Bolsheviks, and for Lenin in particular, to accept the class neutrality of Taylorism and thus of the conveyor belt. More generally, it became possible to accept the neutrality of the productive forces and thus to think that it would have been possible to build socialism by using the *capitalist* productive forces, i.e. productive forces with a capitalist class content. Herein lay the fundamental weakness of the Soviet Union and the ultimate reason of its demise (see Carchedi, 1983, 1987, 2005). The course of history might have been different if the productive forces and thus knowledge had been seen as social phenomena with a definite class, i.e. capitalist, content. But a proper understanding of dialectics is essential also within the present social context. Take, for example, the class content of economic neo-liberalism. Neo-liberalism is based on an economic theory that, in its turn, is based on the notion of homo economicus, an economic agent whose rationality is supposed to be natural, i.e. the manifestation of human nature, rather than being the embodiment of capitalist rationality (see below, the fifth section, *On the Class Determination of Homo Economicus*). Thus, economic policies deriving from the supposed class neutrality of economics cannot but favour Capital at the expense of Labour. The practical, political and social implications of different views on dialectics are thus far reaching and all important.[4]

Formal Logic and Dialectical Logic

Mainstream social sciences make use of traditional formal logic. The question, then, is whether formal and dialectical logic exclude each other or whether they can coexist.[5] Formal logic too rests on three basic laws. The law of identity states that something is equal to itself, i.e. $A = A$. It is well known that this is nothing more than a truism. As such it cannot generate any knowledge about A. The law of the excluded middle states that $A = A$ is either true or not true, i.e. either $A = A$ or $A \neq A$. There is no third possibility. The law of non-contradiction, states that two contradictory propositions cannot both be true. A proposition, $A = A$, and its denial, $A \neq A$, cannot both be true. The focus in what follows will be on the law of identity, given that the other two laws are derived from it.

As just pointed out, $A = A$ is a truism. To be a meaningful statement, it must also be possible for A to be different from A, i.e. $A \neq A$. In this case, we can inquire into the conditions for $A = A$ and for $A \neq A$, i.e. why and how is $A = A$ and why and how $A \neq A$. This is what dialectical logic does. For dialectal logic, A is at the same time both itself (i.e. $A = A$) and different from itself because of both its realized and of its potential nature. Given that both A^r and A^p are two aspects of the same phenomenon, $A^r = A^r$ and at the same time $A^r \neq A^p$.[6] Formal logic is blind to the realm of potentialities so that A^r is always equal to A^r. But, given that things change continuously over time because of their potentialities, change is banned from this view. Or, to focus only on A^r while disregarding A^p, i.e. to state that A is always equal to A, implies a timeless dimension. This allows us to distinguish dialectical contradictions from logical mistakes.

Case 1. Formal logic contradictions. If only realized reality, A^r, is considered, only $A^r = A^r$ holds and $A^r \neq A^r$ is a logical mistake. What has become realized can be only what it is, as a realized phenomenon. An eight-hour working day is just that ($A^r = A^r$) and to assert that a working day is also not an eight hour working day ($A^r \neq A^r$) is a logical contradiction, a mistake. Thus, in the realm of the realized, which is formal logic's only domain, $2 = 2$ and $2 = 1$ is a contradiction.

Case 2. Meaningless contradictions. If we consider both realizations and potentials, [$A^r = A^r$ and $A^r \neq A^p$] is a meaningless contradiction *if A^p is not contained in A^r*. In fact, it is meaningless to assert that a realized phenomenon is different from what it cannot potentially be. For this reason, this type of contradiction cannot explain change by definition. This can be the case either because A^p does not exist in reality or because it is excluded by definition by A^r. Notice that this type of contradiction is meaningless from the standpoint of a theory of (social) change.[7] The contradiction between a *realized* sheep and a *potential* horse is a meaningless contradiction because a horse is not a potential development of a sheep. Or, the knowledge of human metabolism does not contain within itself the potential to evolve into astronomy so that the contradiction between realized medicine and astronomy as a potential development of medicine is meaningless. Or, a realized eight-hour working day cannot be different from a potential 25-hour working day because the latter does not exist, because the same forces that fix the length of the working day at eight hours cannot fix it at 25 hours.

Case 3. Dialectical contradictions. If we consider both the realized and the potential, [$A^r = A^r$ and $A^r \neq A^p$] is *not* a logical contradiction *if A^p is contained in A^r*, if A^p is a real possibility because it belongs to the potential realm of reality contained in A^r. In this case we have a real, or dialectical, contradiction. That a realized eight-hour working day is different from a potential 10-hour working day is a dialectical contradiction because a 10-hour working day is a real possibility, because the same forces that fix the length of the working day at eight hours can also change it to 10 hours, thus explaining (the possibility of) its change. *A dialectical contradiction is a contradiction between what is and what can be(come) as contradictory to what is.* Far from being a logical mistake, a dialectical contradiction is what allows for, and explains, change. In dialectical logic, a temporary lack of change is explained not in terms of lack of movement but in terms of opposing forces temporarily unable to override each other, as for example an unchanged average rate of profit is the result of the tendency being unable to override the counter-tendency (or vice versa). On the other hand, for formal logic *all* contradictions are a mistake.[8] Formal logic cannot explain change. Thus there is no division of labour between dialectical logic (movement) and formal logic (absence of movement). They are incompatible. Formal logic reduces to a succession of static moments what is a view of social reality as being a temporal flow of determining and determined contradictory phenomena continuously emerging from a potential state to become realized and continuously going back to a potential state.

It follows that formal logic, seen from the standpoint of its class content, is an ideology because it rules out dialectical contradictions and thus movement and change. As will be mentioned in the section *On the Class Determination of Homo Economicus* below, an

ideology is a form of knowledge that defends, implicitly or explicitly, the interests of a class as if they were the interests of all classes, usually by denying the existence of classes. This is the case for formal logic as well. It was born in a slave society and was functional for the reproduction of that society. It was a static view of reality, a rationality in which radical change was absent. By extension, it was the status quo that was rational. It continued to be accepted in subsequent societies, including capitalism, because it can perform the same reactionary function in societies which, however different, share the common feature of being class divided societies and in which it is in the interest of the ruling classes to use and foster this implicit rationalization of the status quo. This accounts for the resilience of formal logic which after having been worked out by Aristotle has remained basically the same for over 2000 years (in its traditional version). Formal logic is an ideology not so much because of what it says but because of what it does not say. Those Marxists who accept formal logic as the method of social analysis cannot ground theoretically the analysis of social change. Given that Marx's theory is informed by dialectics, the banning of dialectics cannot but result in a static and thus conservative view. Formal logic and dialectical logic do not complement each other; they exclude each other because of their opposite class content. On this point the present approach differs substantially from that of Engels and of many Marxists after him.[9]

Nevertheless, if the class content of formal logic is the opposite of, and excludes, that of dialectical logic, the principles of formal logic can and should be applied within dialectical logic as an auxiliary method because the rules of formal logic apply to the realm of the realized (which without the potentials is a static reality) and only to that realm. While exclusive focus on the realized disregards the potential and thus cannot account for dialectical contradictions, movement and change (Part I, Section 1), consideration of the realized as a partial step in the analysis is acceptable and necessary if one chooses as a level of abstraction only the realm of the realized within a view of reality stressing both the realized and the potential. The rules of formal logic, if immersed in a dialectical interpretative scheme, do not deny dialectical contradictions, movement and change but complement their understanding. To ban dialectical contradiction, movement and change from analysis (as in formal logic) means to adhere to a specific class content of the analysis. But to temporarily disregard these features of dialectical logic, and thus A^p, to analyse separately $A^r = A^r$ and $A^r \neq A^p$ as a technique within a dialectical framework, is methodologically possible and necessary.[10]

For example, Marx analyses the subdivision of the working day into necessary and surplus labour time by holding the length of the working day constant. The premise is $A^r = A^r$, i.e. eight hours are eight hours. The purpose is to focus on the movement within A^r: the greater the necessary labour time, the smaller the surplus labour time and vice versa. However, a full comprehension of A^r requires an insight into A^p and thus into $A^r \neq A^p$, i.e. into how the same forces that determine the subdivision within a working day of a certain length (A^r) can determine also a potential change in A^r. By considering that a realized eight-hour working day is different from a potential 10-hour working day, we hypothesize the possibility of a change in the length of the working day. This does not negate the results obtained by taking an eight-hour working day as a constant but enriches the

analysis by transforming that constant into a potential variable. Formal logic cannot encompass dialectical logic because the former shuns contradictions, because for it all contradictions are mistakes.[11] But dialectical logic does encompass formal rules of reasoning (but not formal logic with its class content) even though these rules cannot explain dialectical contradictions and thus contradictory change (a change within realized reality as well as a change of realized reality). But there are limits to the application of the rules of formal logic within dialectical logic. Let us consider induction, deduction and verification in the two approaches.

Induction, Deduction and Verification

Deduction can test whether some conclusions follow from some premises. These premises have been arrived at by a previous process of induction. This method applies to both formal and dialectical logic. However, there are fundamental differences. The first concerns the specificity of dialectical induction and deduction.

> It seems to be correct to begin with the real and the concrete, with the real precondition, thus to begin, in economics, with e.g. the population, which is the foundation and the subject of the entire social act of production. However, on closer examination this proves false. The population is an abstraction if I leave out, for example, the classes of which it is composed. These classes in turn are an empty phrase if I am not familiar with the elements on which they rest. E.g. wage labour, capital, etc. These latter in turn presuppose exchange, division of labour, prices, etc. For example, capital is nothing without wage labour, without value, money, price etc. Thus, if I were to begin with the population, this would be a chaotic conception [*Vorstellung*] of the whole, and I would then, by means of further determination, move analytically towards ever more simple concepts [*Begriff*], from the imagined concrete towards ever thinner abstractions until I had arrived at the simplest determinations. (Marx, 1973: 100)

For Marx, then, induction starts with observation, the 'chaotic conception of the whole', e.g. the population. It then finds what is specific to the population, i.e. classes, not by assuming away the population but by *compressing* the population into classes. The notion of classes is less rich in details, 'thinner', but it *contains* within itself that of the population and thus becomes the population's condition of existence, its determinant. The population in its turn becomes the condition of reproduction or supersession of classes. Dialectical induction, thus, proceeds from determined to determinant, it reduces what is realized to a potential state, it discovers the condition of existence of realized reality. In short, *dialectical induction compresses realized reality into its conditions of existence* (it discovers the determinants of realized reality). At each stage in the process of induction the result obtained in the previous stage becomes the starting point for a new step until the simplest determinations are reached. For example,

if population is the starting point of induction, the outcome is a more abstract concept, class. The next step starts from classes (an abstract concept) and works out still more abstract concepts (production and appropriation of surplus value), etc. To abstract does not mean to abstract away from reality, to reach concepts progressively devoid of concrete reality, but to progressively concentrate concrete reality into the concepts of its essential nature as simpler and simpler determinants. The simplest determination reached through dialectical induction, then, contains *in nuce*, as potentials, all other aspects of social reality. As we have seen, for Marx this is the capitalist production (ownership) relation.

At this point induction terminates and deduction begins. The excerpt above continues as follows: 'From there the journey would have to be retraced until I had finally arrived at the population again, but this time not as the chaotic conception of a whole, but as a rich totality of many determinations and relations'. This is what Marx calls the concrete in thought (Marx, 1973: 100–101).[12] The 'retracing' phase is *dialectical deduction*, the unfolding (reconstruction in thought) of more and more concrete, detailed and articulated pictures of reality from their determinant factors. Each step in the unfolding is a (temporary) conclusion but also the premise for the following step in the chain of deductions.

The above raises a further question: which criterion should be used both in the inductive and in the deductive phase. In other words, how can we validate the choice of the determinant aspects inherent in realized reality and of the development of pictures of realized reality from its determinant aspects? Hypothetically, one could use Marx's method (induction and deduction) in ways contrary to Marx's project. For example, one could argue that the potential aspects of the population are groups of mentally inferior and superior individuals. The criterion, not found explicitly in Marx but deeply ingrained in his theory, is that the validity of the concepts reached through the dialectical process of induction and deduction (i.e. through knowledge formation) is assessed *in terms of their social content* and thus, a posteriori, in terms of the social content of the theory built upon them. From the standpoint of Labour, this means that the choice has been valid if those concepts and the theory built upon them are functional for the supersession of capitalism.[13]

A case in point is the choice of the capitalist production relations and the ultimately determinant factor. Is there a justification for Marx's choice or is it arbitrary? Marx again:

> The conclusion we reach [the conclusion of a previous process of induction, G.C.] is not that production, distribution, exchange and consumption are identical, but that they all form the members of a totality, distinctions within a unity. Production predominates not only over itself, in the antithetical definition of production [this is the contradictory nature of the capitalist production relation, see Part I, Section 1, G.C.], but over the other moments as well. The process always returns to production to begin anew [after what has been produced in one period has been distributed, exchanged and consumed, a new production process starts in the following period, G.C.]. That exchange and consumption cannot be predominant is self-evident [see below, G.C.] ... A definite

production thus determines a definite consumption, distribution and exchange as well as definite relations between these different moments [production is the ultimate condition of existence of distribution, exchange and consumption, G.C.]. Admittedly, however, in its one-sided form, production is itself determined by the other moments. For example if the market, i.e. the sphere of exchange, expands, then production grows in quantity and the divisions between its different branches become deeper [exchange, as well as all other determined elements, is a condition of reproduction or supersession of production, G.C.]. Mutual interaction takes place between the different moments. (Marx, 1973: 100)

In what sense, then, does production predominate over distribution, exchange and consumption? In general, because, in all societies, the former must precede the latter and thus informs the latter. Only what comes first can influence what comes later (to hold a contrary view would imply to erase time). Distribution, exchange and consumption can influence production only in the following period. But each society has its own specificity. There is thus a specific sense in which production predominates under capitalism. What is specific to this system is that the producers have been expropriated of the means of production and must sell their labour power to the owners of the means of production. If this is capitalism's specific element, it is also that which informs the rest of society (social phenomena), the determining element in the last instance. It is the contradictory social content of the capitalist production relation that spreads itself to the other social phenomena. But these latter are not simply epiphenomena, mirror images, of the production relation. They come to life in the process of mutual determination. Thus, the specificity of capitalism is not power relations, nor the political, ideological or economic oppression of social groups. This takes place also in other class divided societies even though their socially determined and specific form of manifestation arises in the process of mutual determination with all other social phenomena, including the production relation. The specificity of capitalism is the capitalist ownership relation, something that no matter which form of manifestation it takes cannot be found in any other type of society.[14]

Why, then, is a theory built on the ultimately determining role of the capitalist production relation functional for the supersession of capitalism? It is impossible to abolish exploitation (and the same holds for all forms of discrimination and oppression, economic, ideological and political) *without changing first what is specific to capitalism*, its essence, the production relation. Sexual, racial, etc. discrimination can be abolished and yet capitalism can survive either without them or by generating new forms of discrimination. Exploitation, on the other hand, cannot be abolished without first changing the relation of ownership at the basis of the production relation. Recent history has shown that only a specific conjuncture, as consciously understood by the masses' revolutionary genius, can decide which forms of struggle in which areas of society are sufficiently strong to defeat capitalism (the Leninist weakest link in the imperialist chain), in the sense that, having defeated it, one can start building a new society. But it is one thing to defeat capitalism; another to build socialism. A new phase in human history must start from a specific point, from the transformation of the

production relation and thus of the production process on the basis of solidarity, equality and cooperation. It is the new production relation and process that will irradiate their social content (which is not a class content any more) to all other social phenomena.[15]

The second difference is that in formal logic the premises should not be contradictory. If they are, the conclusions cannot but be ambiguous and undetermined. To return to an example mentioned above, technological innovations can *both* increase the (surplus) value produced (e.g. if they decrease the value of the means of production per unit of capital invested) *and* decrease it (e.g. if they replace people with machines, given that only human labour can produce value and surplus value). From this contradictory premise (the result of previous induction), it is impossible for formal logic to conclude unambiguously whether the new value produced increases or decreases as a result of technological innovations. On the other hand, dialectical logic deduces from this contradictory premise a tendential movement, a movement exhibiting tendencies and counter-tendencies (Part I, Section 1), a contradictory movement. This is not a logical mistake but a rendition of a real movement. The same holds if we start from two mutually contradictory premises. This does not imply that deduction as in formal logic should be discarded. It allows us to distinguish correct from incorrect statements about social phenomena if only non-contradictory premises are considered, i.e. if they are separated from their movement and change, and thus from the contradictory relations with other phenomena. For example, if only the premise of labour reduction is considered, the average rate of profit can only fall as a consequence of 'labour saving' technological innovations. Within this partial approach deduction as in formal logic applies. However, it is only one aspect of the analysis of contradictory movements.[16]

The third difference concerns the choice of the premises. Deduction as in formal logic requires the explicit enunciation of all the premises that are needed in order to necessarily reach an unambiguous conclusion. This is impossible in dialectical logic and more generally in the social sciences because all elements of reality are interconnected.[17] In this case, one has to choose the determinants from a vast array of real contradictory causes, the premises that can explain contradictory movements. Then, one models in thought the real, contradictory and tendential movement. And finally, one decides which of the premises is the tendency and which are the counter-tendencies. If the result explains the movement *in its characteristic features rather than in all its aspects*, the test is successful. The aim is a theory with explanatory power, logically consistent and evidentially right. There will be other factors affecting that particular tendential movement. But they can be ignored if the test is successful in the above mentioned sense. This allows us to forecast the repetition of the movement in its characteristic features *as long as those premises are unchanged*. This answers the objection that it is impossible to know that an event will recur in the future simply because it has taken place in the past. This position makes forecasts impossible in the social sciences.[18]

Just as induction and deduction differ in the two approaches, so does the verification of what has been induced and deduced. In the present approach, verification is both theoretical and empirical. The former concerns whether the new element of knowledge is internally

consistent with the theory within which it has been generated. Consistency refers both to formal logic, whenever applicable, and to dialectical logic. In terms of this work, this latter means that the new element of knowledge must be *consistent with the social content* of the more general theoretical framework. Theoretical verification in terms of social content is generally disregarded due to the myth of the neutrality of knowledge. A test of theoretical consistency of an element of knowledge within a broader theoretical frame fails if the former carries a social content different from that of the latter. A pertinent case is the substitution of the premise from which some results have been deduced with another premise with a different social content. For example, Okishio's critique of Marx's law of the tendential fall of the profit rate is invalid because that critique is based on a notion of labour as a cost, while for Marx labour is indeed a cost for the individual capitalists but it is also and above all the only value creating activity. The expulsion of labour by a capitalist is indeed cost reducing for that capitalist but it decreases at the same time the (surplus) value produced, thus decreasing the average rate of profit. The former notion of labour reflects the interests of Capital, the latter those of Labour (see Carchedi, unpublished B).

Empirical (evidential) verification refers to the empirical consistency of factual data with the knowledge being tested. Empiricism holds that theories should be tested on the basis of neutral data. However, neutral data, in the sense of being worked out outside theories and thus with no social content, do not exist simply because all elements of knowledge have a social content due to their dialectical determination. The empiricist illusion arises because different theories can use the same quantitative methods and collect the same quantitative data, e.g. the number of people out of work. Then, neutrality of data would seem to derive from the neutrality of mathematical and statistical methods as shown by their use by different theories. However, the simple act of counting (the unemployed) is meaningless. Counting has a meaning because of a certain view of reality (theory) with a certain social content that requires that counting. Then, if a theory uses a certain quantitative method it transfers to it its social content. The same data are the same quantitatively, but qualitatively (in terms of social content) different. The point is not that the same data are interpreted differently within different theories, e.g. that data on the unemployed can be defined as full employment or unemployment. On this there is general agreement. Rather, the point is that those quantitative methods acquire a different social content according to the theory within which they are employed (they are meaningful only within a theory so that their meaningfulness depends on which theory employs them), so that the same holds for those data in their 'pure' quantitative form.

Marx's Mathematical Manuscripts

Consideration of the relation between empirical, i.e. quantitative, and theoretical, i.e. qualitative, verification raises the question of the relation between mathematics and dialectical logic. Usually commentators focus on Marx's *Mathematical Manuscripts* (Alcouffe, 1985, 2001: 142–65; Antonova, 2006; Blunden, 1984; Engels, 1983: xxix, 1987; Gerdes, 1985; Kennedy, 1977; Lombardo Radice, 1972; Smolinski, 1973; Yanovskaja, 1969,

1983; Zelený, 1980). As is well known, Marx embarked on the study of mathematics because, as he himself said, his knowledge of algebra was insufficient for his elaborations of the principles of economics.[19] The first evidence of Marx's interest in mathematics is contained in a letter to Engels of 1858 in which he wrote: 'In working out economic principles I have been so damned delayed by mistakes in computation that out of despair I have begun again a quick review of algebra. Arithmetic was always foreign to me. By the algebraic detour I am shooting rapidly ahead again.' (Marx, 1858) By 1863 he wrote to Engels: 'In my free time I do differential and integral calculus'. (Marx, 1863) Most interestingly, in another letter to Engels ten years later (1873), he provides an example of what economic principles he had in mind:

> I have been telling Moore about a problem with which I have been racking my brains for some time now. However, he thinks it is insoluble, at least *pro tempore*, because of the many factors involved, factors which for the most part have yet to be discovered. The problem is this: you know about those graphs in which the movements of prices, discount rates, etc. etc., over the year, etc., are shown in rising and falling zigzags. I have variously attempted to analyse crises by calculating these ups and downs as irregular curves and I believed (and still believe it would be possible if the material were sufficiently studied) that I might be able to determine mathematically the principal laws governing crises. As I said, Moore thinks it cannot be done at present and I have resolved to give it up for the time being. (Marx, 1873)

In terms of the present approach, 'to determine mathematically the principal laws governing crises' is an impossible task. First, mathematics is a branch of formal logic and premises in formal logic cannot be contradictory. However, to account for the laws of movement one has to start from contradictory premises (in the sense of dialectical contradictions) and this is why, as mentioned above, the laws of movement are tendential. Second, even if all the 'factors involved' were known, it would be practically impossible to consider all of them. This is why econometric models, even large ones involving thousands of relations, have such a dismal record as tools of prediction.[20]

As mentioned above, dialectical logic singles out *some* contradictory premises, those considered to be the most pregnant determinants, and develops them to reach increasingly detailed depictions of reality. In this way, the *basic* features of the movement, the interplay of the tendency and the counter-tendencies, are depicted and can be forecast irrespective of temporary deviations from the tendential and the counter-tendential movement ('the rising and falling zigzags'). To argue that we should be able to explain and forecast a tendential movement in all its details is to suppose that we could take all possible causes of that movement into consideration. This is tantamount to renouncing any explanation of change. But if it is impossible to determine the laws of crises purely in terms of mathematics, it is certainly possible to analyse the cyclical movement of economic indicators (the ups and downs) by using 'higher mathematics'. This was Marx's intuition and probably this is why he applied himself to the study of calculus.

Two questions arise. First, why did Marx make no use of differential calculus in his work? According to Smolinski, for Marx:

> the key fact is that a commodity has value or does not have it, labor is productive or is not, a participant in the economic process is a capitalist or a proletarian, society is capitalist or socialist. For this polarized universe a binary calculus might be a more suitable tool than differential calculus. (1973: 1199)

However, Alcouffe remarks that the reproduction schemes and the tendential fall of the profit rate are amenable to be treated with the mathematics developed by Marx. For example, differential calculus can be used to compute the instantaneous rate of change in the profit rate (1985: 37). Both opinions seem to have an element of truth. Differential calculus is indeed applicable to some aspects of Marx's economic theory but the question is whether this would be relevant at all. The relevant question is not how the rate of profit changes instantaneously but how it changes due to the dialectical interplay between the tendency and the counter-tendencies.[21] A more probable explanation is that, given that Marx finally mastered calculus towards the end of his life, he did not have the time and opportunity to write an analysis of the quantitative aspects of economic life (for example, of the economic cycle, the 'zigzags' as he puts it in the letter above).

The second question is how Marx would have applied calculus had he had the time and opportunity. This question cannot be settled by considering how mathematics has been applied in economic planning by formally centrally planned economies. As Smolinski reports, 'According to a widely held view, it was Marx's influence that has delayed by decades the development of mathematical economics in the economic systems of the Soviet type, which, in turn, is said to adversely affect the efficiency with which they operate.' (1973: 1189) But, as the author rightly points out and as the *Manuscripts* show, Marx was far from being ignorant of calculus and was greatly interested in its application to economics. It is true that

> The planners' 'mathematicophobia', to use L. Kantorovich's apt expression, led to a substantial misallocation of resources through nonoptimal decisions … The intellectual cost of the taboo in question was also high: reduced to a status of a 'qualitative,' dequantified science, economics stagnated … [Oskar Lange] pointed out that Soviet economics degenerated into a sterile dogma, the purpose of which became 'to plead the ruling bureaucracy's special interests and to distort and falsify economic reality.' These processes led to a withering away of Marxism ... Marxist [economic] science was replaced by a dogmatic apologetics. (Smolinski, 1973: 1189)

There is considerable confusion here. While Marx cannot be held responsible for the insufficient application of mathematics in Soviet type economies and while this insufficiency was certainly an obstacle to the efficient functioning of an economic system, the reasons for the demise of the USSR and other Soviet-type centrally planned economies should be sought elsewhere. In short, in spite of its specific features, including the absence

of the market, the USSR was already a sort of capitalist system where the political/managerial class was performing the function of capital. The application of planning techniques was meant to mirror the market as an allocation system. It was thus opposite to a system based on the labourers' self-management of the economy and society. Contrary to Smolinski's view, the planners' choices were often mistaken not because they 'reflected the mistaken labor theory of value' (1973: 1190) but because an inherently capitalist system needed the market as an allocation system rather than any other type of system. The optimal allocation *for capital* can only be through the market. The system was thus inherently weak and unable to compete with fully developed capitalist systems (Carchedi, 1987).

As for Marx, the question here is not whether and how the would have applied differential calculus to his economic theory. This is of scarce importance. Rather, the point is that even though the *Manuscripts* do not deal with the relation between dialectics and differential calculus, Marx's *method of differentiation* provides key insights into Marx's notion of dialectics. This point has escaped all the commentators of the *Manuscripts*. Yet, it is these insights rather than Marx's own original method in dealing with calculus that are the really important aspect of the *Manuscripts*.

Let us begin by considering how 'Leibniz arrived at the notion of derivative … from geometric considerations' (Gerdes, 1985: 24; see also Struik, 1948: 187ff). Let $y_1 = x_1^3$. Starting from $dx = x_1 - x_0$ and $d_y = y_1 - y_0$,

(1) $y_1 = x_1^3 = (x_0 + dx)^3 = x_0^3 + 3x_0^2 dx + 3x_0 (dx)^2 + (dx)^3$

Given that $y_0 = x_0^3$

(2) $y_1 = y_0 + 3x_0^2 dx + 3x_0 (dx)^2 + (dx)^3$

so that

(3) $y_1 - y_0 = dy = 3x_0^2 dx + 3x_0 (dx)^2 + (dx)^3$

and dividing both members by dx we obtain

(4) $dy/dx = 3x_0^2 + 3x_0 dx + (dx)^2$

At this point, following Leibniz, we can cancel dx on the right given that dx is infinitely small. Thus, we obtain

(5) $dy/dx = 3x_0^2$ or more generally $3x^2$ (Gerdes, 1985: 24–30).

The problem according to Marx is twofold. First, the derivative $3x_0^2$ already appears in equation (1), i.e. *before the derivation*, before dx is set equal to zero. Thus, to get the derivative, 'the terms which are obtained in addition to the first derivative $[3x_0 dx + (dx)^2]$ … must be *juggled away* to obtain the correct result $[3x_0^2]$' (Marx, 1983: 91). This is necessary 'not only to obtain the true result but any result at all' (1983: 93). Marx calls this the 'mystical' method. Second, if dx is an infinitesimally small quantity, if it is not an ordinary (Archimedean) number, how can we justify the use of the rules for ordinary numbers, e.g. the application of the binomial expansion to $(x_0 + dx)^3$? More generally, what is the theoretical status of infinitesimally small quantities?

In dealing with these difficulties, Marx develops his own method of derivation. Basically, Marx's method is as follows. Given a certain function, such as $y = f(x)$, Marx first lets x_0 become x_1. Both x and y increase by a *finite* quantity, Δx and Δy (so that the rules for ordinary numbers can be applied here). The ratio $\Delta x/\Delta y = [f(x1)-f(x0)]/(x1-x0)$ is what he calls the provisional or preliminary derivative which is the limit of a ratio of finite differences. Then, he lets x_1 return to x_0 so that $x_1-x_0 = 0$ and thus $y_1-y_0 = 0$ thus reducing this limit value to its absolute minimum quantity. This is called the definitive derivative, dx/dy (so that the derivative appears only after the process of differentiation).[22] 'The quantity x_1, although originally obtained from the variation of x, does not disappear; it is only reduced to its minimum limit value = x' (1983: 7). Let us then see how Marx computes the derivative of $y = x^3$.

If x_0 increases to x_1, y_0 increases to y_1. Given that $x_1-x_0 = \Delta x$ and $y_1-y_0 = \Delta y$

(1) $\Delta y/\Delta x = (y_1-y_0)/(x_1-x_0) = (x_1^3-x_0^3)/(x_1-x_0)$

Given that

(2) $(x_1^3-x_0^3) = (x_1-x_0)(x_1^2 + x_1x_0 + x_0^2)$

we substitute (2) into (1)

(3) $\Delta y/\Delta x = [(x_1-x_0)(x_1^2 + x_1x_0 + x_0^2)]/(x_1-x_0)$

and we get the provisional derivative

(4) $\Delta y/\Delta x = x_1^2 + x_1x_0 + x_0^2$

To get the definitive derivative, x_1 goes back to x_0 so that $\Delta x = dx = 0$ and $\Delta y = dy = 0$. Equation (4) becomes

(5) $dy/dx = x_0^2 + x_0^2 + x_0^2 = 3x_0^2$

The definitive derivative is thus the 'preliminary derivative reduced to its absolute minimum quantity' (Marx, 1983: 7). The two methods conduce to the same results. But this is just about the only thing they have in common. First, 'the starting points … are the opposite poles as far as operating method goes' (1983: 68). In one case it is $x_0 + dx = x_1$ (the 'positive form'); in the other (Marx) it is x_0 increasing to x_1, i.e. $x_1-x_0 = \Delta x$ (the 'negative form') (1983: 88). 'One expresses the same thing as the other: the first negatively as the difference Δx, the second positively as the increment h' (1983: 128). In the positive form 'from the beginning we interpret the *difference* as *its* opposite as a *sum*' (1983: 102). Second, the procedures differ too: the fraction $\Delta y/\Delta x$ is transformed into dy/dx (i.e. we start from finite quantities that we subsequently set equal to zero) and the derivative is obtained after the derivation, after dx is set equal to zero. In the positive method (form) 'the derivative is thus in no way obtained by differentiation but instead simply by the expansion of f(x+h) or y_1 into a defined expression obtained by simple multiplication' (1983: 104). Third, the interpretations too are radically different. Marx's procedure allows him to realize that dx/dy is not a ratio between two zeros but a *symbol* indicating the procedure of first increasing x_0 to x_1 (and thus y_0 to y_1) and then reducing x_1 (and thus y_1) to

their minimum values, x_0 and y_0. Marx's discovery that dx/dy is an operational symbol anticipated 'an idea that came forward again only in the 20th century' (Kolmogorov, quoted in Gerdes, 1985: 75). Marx's stress on dx/dy as being an operational symbol, the 'expression of a process' (Marx, 1983: 8), the 'symbol of a real process' (1983: 9), is a real achievement, 'an outstanding critique of the "mystical" foundations of infinitesimal calculus, of the metaphysical nature of infinitely small entities which are neither finite not null' (Lombardo Radice, quoted in Ponzio, 2005: 23). For Marx, Δx does become equal to zero. If Δx and Δy become zero, they become dx and dy and $dy/dx = 0/0$. But this is just a symbolic notion, an operational symbol, for the definitive derivative. It is a symbol of a process, of x_0 first increasing to x_1 and then going back to x_0.

From a mathematical point of view, Marx's method is of limited applicability 'because it is often impossible to divide $f(x_1)-f(x_0)$ by x_1-x_0' (Gerdes, 1985: 73).[23] Nevertheless, in spite of its computational limits, this method offers important insights into Marx's notion of dialectics.[24]

First, for Marx, a quantity, x, can be either x_1 or x_0. The notion of an infinitesimally small quantity, of an infinite approximation to zero, of something that as a realized entity is neither a number nor zero, should be rejected as 'metaphysical', as a 'chimera'. In the realm of realized reality a quantity cannot be at the same time zero and different from zero. Only if the realm of the potentials is considered a quantity (e.g. GNP) can it be both what it actually is and potentially another quantity or even zero. The same point is made by Yanovskaya, as reported by Gerdes: 'some scientists explained the infinitesimals or infinitely small quantities in terms of the dialectical nature of opposites – at the same time equal to zero and different from zero. Yanovskaya called these scientists "*pseudo-Marxists*" because they forgot that dialectical materialism does not recognize static contradictions ($=0$ and $\neq 0$), but only contradictions connected with motion' (Gerdes, 1985: 115–16). This is consonant with the present approach in the sense that contradictions connected with motions are the contradictions between potentials and realized. Given that dx indicates changes in x that, however small, belong to *realized* reality, dx cannot be zero and different from zero at the same time. This is a formal contradiction, a mistake. To call this a dialectical contradiction is simply to paper over an unsolved problem and to do dialectics a disservice. One thing is to make x first grow from x_0 to x_1 and then to let it go back to x_0, i.e. to reduce it 'to its absolute minimum quantity'. This is the smallest variation and yet it is an actual variation, a variation in the realm of the realized. Another is to consider dx as something that is both an actual variation and no variation at all (zero). If x goes from x_0 to x_1 and back to x_0 (an actual variation) it is because x_0 as the point of arrival was already potentially, implicitly present in x_0 as the point of departure. Marx's discussion of the derivative supports indirectly that aspect of dialectics submitted here that distinguishes between realized and potentials.[25]

Second, $x_0 + dx$ indicates an addition, a variable (dx) added to a constant quantity (x_0). Implicitly, x_0 remains constant throughout, so that movement and change affect only a limited section of reality.[26] The starting point is a constant, a lack of movement and of change, to which change is added only as an appendix. If $dx = 0$, change stops and the situation reverts to stasis. This is a view of a static reality only temporarily disturbed by a movement that moreover applies only to an infinitesimal part of reality. The analogy

with equilibrium and disequilibrium (temporary deviations from equilibrium) in the social sciences is clear. On the other hand, for Marx 'x_1 is the increased x itself; its growth is not separated from it … This formula distinguishes the increased x, namely x_1, from its original form prior to the increase, from x, but it does not distinguish x from its own increment' (1983: 86). In other words, x_0 cannot increase by Δx (or dx) without changing into x_1; the change in a part of reality (however small) changes the whole of it due to the interconnection of all of reality's constituent parts. The movement from x_0 to x_1 (Marx's starting point) and back (the end point) indicates a change in the whole of reality, even if caused by a minimal part of it. This is a dynamic view in which absence of movement and change play no part. Again, computationally, both procedures arrive at the same result. But the interpretation differs diametrically. Behind the former lies a static interpretation of reality, behind the latter a dynamic view. Since movement and change in the real world imply time, they imply a movement from x_0 to x_1 at time t_1 and back to x_0 at time t_2. Marx's method of differential calculus is consonant only with a dynamic and temporal approach (and inconsistent with an approach in which time does not exist, as in simultaneism) and more generally with the notion of dialectics submitted here. This conclusion is of the greatest importance when it comes to issues of political economy, as for example in the recent discussions on the so-called transformation problem and on the law of the tendential fall in the rate of profit.

Third, Marx mentions in passing and only once that his two-step approach to the derivative is an example of the negation of the negation: 'The whole difficulty in understanding the differential operation (as in the negation of the negation generally) lies precisely in seeing *how* it differs from such a simple procedure and therefore leads to real results.' (Marx, 1983: 3) This seems to be a fairly thin basis for arguing that 'Marx is interested above all to show how the negation that is at work in differentiation, if rid of its metaphysical approach, turns out to be a dialectical negation' (Ponzio, 2005: 33; see also Kennedy, 1977: 311). Alternatively, one would be tempted to consider it as an example of Marx coquetting with the mode of expression peculiar to Hegel (see note 2 in Part I). But Marx wrote that he had been coquetting with Hegel's terminology in 1873 while the *Manuscript* under consideration was written in 1881. Thus it would seem that Marx was really thinking of this as an example of the negation of the negation.

In this connection, it should be mentioned that 'the way Marx expresses himself in the Manuscripts … reflects the notion largely accepted in his time according to which numbers, if subjected to mathematical operations, change, i.e. grow, diminish, become null, etc.' (Ponzio, 2005: 27). If this is the case, this 'law' differs significantly from the same law in the social sciences. In fact, this movement is:

1) only a quantitative change from x_0 to x_1 and vice versa, i.e. there is no qualitative change; and:
2) no account is given of the forces inherent in x_0 that cause it to change to x_1 and vice versa.

The account of the social forces determining qualitative changes is specific to the social sciences and accounts for the possibility for social phenomena to supersede themselves.

We thus return to the point made above in discussing Engels. Whether or not there is a dialectics in nature, i.e. whether or not the laws of dialectics are 'valid just as much for motion in nature and human history and for the motion of thought' (quoted in Gerdes, 1985: 88), the dialectical method differs substantially in the analysis of natural from that of social phenomena.[27]

Aside from the relation between calculus and dialectics, some further remarks can be made concerning the relation between mathematics in general and dialectics. First, for Marx, 'the algebraic method … [is] the exact opposite [of] the differential method' (1983: 21), because the former is the analysis of static quantities while the latter analyses changing quantities. However, in spite of this difference, both branches of mathematics share a characteristic: that of dealing only with quantities and thus with quantitative change. If mathematics deals only with quantity, it deals only with the realm of realizations (i.e. not with the realm of potentials). It follows that what was said above about the relation between formal and dialectical logic applies also to that between mathematics and dialectics. First, given that contradictions in the realm of realized reality can only be formal logic contradictions (i.e. logical mistakes), mathematics excludes contradictions which it perceives only as logical mistakes. By not dealing with the realm of potentials, it cannot deal with dialectical contradictions (contradictions between the realized and the potential aspects of social reality) and thus with qualitative change. Like formal logic, it too cannot theorize change in the social world, it can only quantify some aspects of change.[28]

It follows that, as in the case of formal logic, the *tools* of mathematics are fundamental auxiliary tools in social research if, and inasmuch as, analysis of social reality allows for quantification. But its application to unquantifiable concepts leads to nonsense. For example, marginal utility (du/dx), upon whose quantification the whole marginalist theory is based, cannot be quantitatively compared between individuals because it is by definition an individual psychological notion. Abstract labour (and thus – indirectly – value), on the other hand, can be measured because its physical dimension is the expenditure of abstract human energy (measurable for example in kcalories). The reduction of social analysis to only those phenomena that are quantifiable leads to an approach with an anti-dialectical and thus anti-Labour class content (see Carchedi, unpublished A).

Finally, even though mathematical concepts can be derived from a specific section of concrete reality, they can be applied to *any* realm of *quantifiable* reality. Mathematics abstracts away concrete reality. For this reason, it does not seek confirmation in the real concrete. On the other hand, dialectical logic is a theoretical concentrate of concrete reality. For this reason it seeks validation in that reality.

On the Class Determination of Homo Economicus[29]

A theory focused on a non-equilibrium view of a society tending towards its own supersession (or even destruction) must reckon with the most powerful contrary view, neo-classical economic theory. Not only is this the only theory that explicitly theorizes the reproduction of the economy through the equilibrating function of the market and its optimal allocation of

resources, it is also, this is the most pervasive theory in the social sciences. It has percolated into all spheres of social consciousness and into everyday perception.[30] It is the exact opposite of the dialectical view developed in this work. This last section aims at examining the foundation of neo-classical economics, economic man and the rationality moving him. It is this rationality that supposedly lies at the heart of the tendency for capitalism to reproduce itself.[31] This rationality will be shown to be both logically faulty and reflecting the standpoint of Capital.[32] Thus, this section provides an example of the social content of a theory that borrows its ideological power from its ability to present itself as socially, class, neutral.

According to neo-classical economic theory, the capitalist economy, if let free to function unencumbered, tends naturally towards equilibrium. This view is based on the following three assumptions. First, the basic unit, and thus the starting point, of analysis is the individual as a unique individual (concrete individuals, in the terminology of this article); second, individuals are equipped with an inborn, a-historical, egoistic rationality; and third, the free exercise of this rationality results in the economy either being in, or tending towards, equilibrium. This presupposes *economic man*, the rational being par excellence. This rationality can be exemplified by the demand and supply curves: if the demand for a good rises, its price rises and vice versa; if the supply of a good rises, its price falls and vice versa. It follows that the two curves have different slopes: one has a rising slope, the other a falling slope. Therefore, they can intersect, thus fixing the equilibrium price, i.e. the price at which demand and supply coincide. At this point, the markets clear and resources are optimally allocated.[33] This is the alpha and omega of economic man's rationality. It is on this rationality that the greatest majority of economic theories are based.

This approach is untenable for a variety of reasons. Consider the demand curve. To draw this curve, the prices at which those quantities are demanded must be fixed beforehand. The same for the supply curve. This is fixed at the intersection of the two curves, i.e. at that particular price, demand equals supply. However, since all prices and quantities are postulated beforehand, the equilibrium price and quantities are thus postulated beforehand as well. The theory is circular (Carchedi, 2006: 50).

Aside from this critique, it is the notion of rationality hidden behind these curves that should be subjected to criticism. The criticism has not always been accurate. It has been said that this rationality is egoistic because individuals maximize their own welfare independently of other individuals, as if they were monads for whom society does not exist (Anderson, quoted in Tsakalotos, 2004: 142). This is incorrect. Economic man is an egoist because he is rapacious and exploitative, qualities which do presuppose the existence of other people as well as his interest for those people. Let us take only one example, the behaviour behind the demand curve. If the demand for a good rises, i.e. if those who do not have that good have a greater need for it, those who do have that good take advantage of the situation (the greater need) and raise that good's price. Moreover, only the purchasers able to allocate a greater purchasing power to these goods will be able to satisfy this increased need. But this is of no concern to the seller. Similarly for the supply curve. If supply increases, prices decrease, i.e. if the sellers need to sell more, the purchasers take advantage of this greater need by offering less. Moreover, the purchasers are

indifferent as to who, among the producers, will be able to sell their commodities and who will face economic hardship because of their inability to sell their product. Thus, the shape of the demand curve reveals the sellers' egoism while the shape of the supply curve reveals the buyers' egoism. Economic man maximizes not independently of others but at the cost and with no concern for the fortunes of others. It is on this basis that markets are supposedly cleared, i.e. resources are optimally allocated. Neo-classical equilibrium implies an egoistic rationality and the allocation of resources is optimal only for those who dispose of the needed purchasing power, and thus not for the greatest majority of the world population. This is a crucial point. If the tools, rather than their use, are inherently biased, the notion of equilibrium cannot but reinforce the perception that it is rational to be selfish.

However, economic agents can and do behave altruistically. Neo-classical economists refute this critique by arguing that individuals do not give up their egoistic rationality if they maximize their pleasure by acting altruistically. But, aside from the obvious ideological advantage for Capital of subsuming altruism under egoism, there are at least two reasons for rejecting this option. First, if human beings are both egoistic and altruistic, if they maximize their welfare by behaving both egoistically and altruistically and if this maximization is rational, then they are *always and by definition rational*. If there is no irrationality any longer, the notion of rationality becomes redundant. Secondly, a behaviour which can be both altruistic and egoistic is inconsistent with neo-classical economics. An altruistic behaviour would lower (rather than increasing) prices if needs (demand) rise in order to make possible the satisfaction of those greater needs. An egoistic behaviour would cause an opposite price movement. But this implies that the demand curve can have both an upward and a downward slope. The same applies to the supply curve. There is no guarantee any longer that demand and supply intersect. Altruism, even if only a sub-case of egoism, is logically inconsistent with neo-classical equilibrium.

Consider now the empirical evidence that human beings actually behave like economic man would have us believe. The demand for a whole range of goods, such as financial goods and status goods, can rise if their price rises and fall if their price falls. While the latter category might be quantitatively relatively unimportant, the same cannot be said of the former. Already in the 1990s, the financial markets were 50 times bigger than the export of goods and services. Moreover, demand is strongly influenced by the economic cycle. In the upward phase, the purchase of means of production, of labour power and of consumption goods increases even though their prices rise. In the downward phase, their demand can decrease even if prices fall. Demand behaves either as presupposed by economic man or not. Economic man is an unpredictable being.

But there is more. The slope of the demand curve, and thus the rationality of economic man, can be empirically tested only if time is banned from analysis. If the question is the change in a person's demand for a good as a result of a price change, it must be assumed that that person's preference for that good is the same after as before the price change. Preferences must not change in that time period, or it will be impossible to test whether and how demand changes as a result of *only* that price change. But, given that preferences might change within a certain time period, the hypothesis can be tested only

if we assume a reality without time. If the theory must be testable, it has to rule out time and it becomes irrelevant. If time is introduced in the analysis, the theory ceases to be testable. In terms of Popperian methodology – to which this equilibrium economics adheres – the theory is pure metaphysics.

More yet. Economic man is not only egoistic and rapacious, he is also snooty. He asserts that his rationality is nothing less than the manifestation of human nature. The hypothesis that human nature is egoistic is based on a skilful ideological move. Consider marginalist theory. This theory rests on the notion of decreasing marginal utility, i.e. the decreasing satisfaction the consumer derives from the consumption of an extra unit of a certain good. In equilibrium, the ratio between marginal utility and price must be equal for all goods. Then, if the marginal utility of a good rises, more is demanded of that good. But at the same time the ratio between that marginal utility and that price rises too. To re-establish equilibrium, its price must also rise. The same for a fall in marginal utility. Three features stand out. First, demand hinges upon the utility derived from the consumption of one extra unit of a good, i.e. demand falls because of increased satiety (even if satiety increases at a decreasing pace). Second, a comparison between utilities implies that they are related to prices, i.e. prices indicate neither purchasing power nor intrinsic value but are simply the factor which makes comparison possible. Third, it is on the basis of the comparison between these ratios that both the shape of the demand curve (i.e. the egoistic and rational nature of economic agents) and equilibrium are founded.

This theory has a definite ideological content. An *ideology* is a form of knowledge that, whether internally consistent or not, defends, implicitly or explicitly, the interests of a class as if they were the interests of all classes, usually by denying the existence of classes. This is the case of marginalism as well. First, if we assume that demand decreases with increasing satiety, the movement of demand is biologically, rather than socially, determined. It is on this trait of human biology that the egoism inherent in the demand curve is founded. But there is an alternative explanation for this behaviour: demand decreases with increasing prices because purchasing power decreases (Linder, 1977: 120). In this case the movement of demand is socially determined. The former explanation reflects the view of those very few for whom purchasing power is no problem (for whom the limit to their consumption is only their satiation), rather than the view of by far the greatest majority of the world population whose purchasing power is insufficient, even though the quantification of 'insufficient' varies temporally and geographically. Marginalist theory superimposes the view of the rich, and thus of capital, upon that of the rest of the world's population. The power of this ideology is that it tells a lie which seemingly corresponds to our daily experience.[34]

Second, if rational behaviour (as depicted by the demand and supply functions) is biologically determined, and if this behaviour is exploitative and egoistic, the economic system most suitable to human nature is one based on egoism and exploitation: the capitalist system. Not only are previous systems declared to be inferior to capitalism; also, a possible future system based on cooperation, solidarity and equality is declared to be irrational because contrary to this supposed human nature. But if one acknowledges that economic man is a creature of the capitalist system and reflects the essence

of this system, the system's legitimacy in terms of its correspondence with human nature vanishes. Economic man, as the synthesis of human rationality, turns out to be an ideological construction. Even such an apparently harmless choice as the use of the Latin term has an ideological flavour because it suggests that the last stadium of human evolution is not Homo Sapiens but Homo Economicus.

Capitalist economic rationality can be defended on other grounds, for example because of its economic success. The systematic generation of economic crises, wars, poverty and deprivation for the greatest majority of the world population, the destruction of our environment, dictatorial regimes and fake democratic regimes, just to mention a few items on the list of capitalism's horrors, make this claim sadly ridiculous.[35] Given that that theory can be defended neither on theoretical grounds nor on grounds of its realism, it is submitted that the theory's strength resides in its forecasting power. This is not only a recognition of the unrealism (and thus impotence) of the theory, it is also a methodological own goal. The forecasting power of economic man is null. As seen above, economic man forecasts the behaviour of economic agents in case they so behave and does not forecast such behaviour in all other cases. These latter are so many and so important that they could just as well be the rule rather than the exception. But this is not the main point. The point is that, even if the forecast is correct (e.g. a price rise as a consequence of a rise in demand), economic man has no valid explanation of it because his rationality is vacuous and self-contradictory. Successful forecasts are then chance events.

In short, economic man, aside from his self-contradictory and ideological nature, lives in a timeless world populated only by selfish human beings. His existence is irreconcilable with the existence both of time and of alternative behaviours. If time and alternative behaviours are introduced as a more detailed analysis, economic man disappears. The theories based upon this cardboard character cannot but be separated from reality by an unbridgeable gap. This is fine if economists and other social scientists want to amuse themselves with models of virtual reality. But if the object of inquiry is real, rather than virtual reality, an alternative approach is needed.[36] This article has submitted just such an alternative.

Notes

1   Part I of this article appeared in the previous issue of Critical Sociology 34/04. In what follows, the two parts will be referred simply as Part I and Part II.

2   However, Engels discusses almost exclusively the natural sciences, a practice followed by commentators up to the present day. See Kircz (1998) for a well-balanced assessment of Engels. Engels has been criticized because he did not consider the most advanced developments in the natural sciences of his time. The critics, however, should show that had his knowledge been more up to date he would have had to reject or modify his notion of dialectics. This is highly doubtful. Lacking this proof, the critique is irrelevant.

3   The advantage of grounding dialectics in nature was that socialism became grounded in the objective laws of nature. On the other hand, placing dialectics outside nature was, and is still, seen as subjectivism and thus an individualistic approach (see for example Gerdes, 1985: 122). But there is also

a third option, dialectics as a class determined method of analysis. To support the thesis of this work, let me only cite the opinion of two physicists. Physics studies objective reality. 'But certain directions are triggered and ideas are formed under the influence of a world view.' (Kircz, 1998: 73) For Baracca: 'Modern science has adopted, applied and developed in extremely efficient ways the fundamental attitude of the relation with nature that characterizes capitalist society; in other words, the exploitation of natural resources, the artificial transformation of nature according to the needs of capitalist production, of profit, of the market.' (2000: 171–2)

4   The theoretical ramifications are equally important. For example, it is on the basis of this notion of dialectics that it can be shown that there is no inconsistency in Marx's transformation procedure, i.e. that the so-called transformation problem is non-existent. See Carchedi, 1996; Carchedi and de Haan, 1996.

5   This section will not review modern types of logic and compare them to dialectical logic (with the exception of Zelený, see note 10 below, and Bradley and Swartz, 1979). This would be the task of a much larger work. Rather, emphasis on traditional logic suffices because of two reasons. First, this is the logic used in the social sciences. Second, the conclusion will be reached that, while dialectical logic can accommodate contradictions in a constructive way, this is impossible in formal logic. For example, 'if any proposition ascribes truth to both members of a pair of contradictories, then that proposition is one which has a contradiction within itself' (Bradley and Swartz, 1979: 18). This applies both to traditional and to modern formal logic. Thus, in what follows, by formal logic it will be meant traditional formal logic.

6   Blunden (1984) stresses the interest shown by Marx in the *Mathematical Manuscripts* 'in the differing roles of the left and right sides of the equals sign … [and that Marx] had previously studied the equivalence relation … in relation to the exchange of commodities'. The author draws the conclusion, concerning the law of identity, that 'this law is a useless tautology which leads nowhere – except in so far as "A on the left" is not the same, but the opposite of "A on the right"'. This would be 'the meaning of the law is the identity of opposites – the statement that every single concept contains two opposite sides'. But Marx examines the exchange of two real and different use values, A and B, and argues that they are equal only because of a third factor, because they contain the same substance, (exchange) value.

7   This type of contradiction can of course be used for other purposes, as for example in science fiction literature.

8   This is different from saying that something can both be (exists) and not be (does not exist). This is not dialectical logic but absurd nonsense deriving from disregarding the existence of the potential.

9   For Engels, 'metaphysical categories', which for him are the same as the categories of formal logic, 'retain their validity' but only 'for everyday use, for the small change of science' (Engels, 1987: 494). This notion is unsatisfactory not only because it does not provide a clear-cut principle as to when dialectical logic is applicable as opposed to formal logic. Also, and more importantly, formal logic explains neither everyday change nor the small change of science.

10 A similar conclusion is reached by Zelený in his analysis of the relation between modern formal logic and dialectical logic: 'In Marxian analysis, elementary induction and deduction … play a legitimate role in so far as one is entitled and required by the relative stability of the essence and the universal to treat that stability as fixed within certain limits' (1980: 100).

11 It would seem that Marx engages in what are nowadays called controlled experiments, holding some variables constant and letting others vary, thus accepting this positivistic method (Paolucci, 2001: 92). But even if the technique is the same, the conceptualization behind it is different. The positivist engaging in controlled experiments thinks she can explain the movement of those variables without resorting to the realm of potentials. But these proportions can change because this is already a real

potentiality contained in the working day. These variations, this change, between realized phenomena could not take place if this were not a real possibility.

12 For a theory of knowledge along these lines, see Carchedi (2005).

13 This, the class specific nature of knowledge formation, is the basic difference between the present approach and the 'reflection theory of knowledge' in which knowledge is simply a translation or even a mirror of the material world into the human brain. However, there is no absolute relativism here, the denial that it is possible to know objective reality. Each class (and other social group) knows the same social reality even if from its own socially determined perspective. For example, different classes (class-determined theories) agree that some people are out of work. The definition, methods of data collection and theories explaining this phenomenon vary to the extent that for some this is unemployment and for others (below a certain level) full employment. But all agree that in reality some people are out of work; otherwise there would not be those different views of that same social phenomenon.

14 In commenting on the above quotation, Resnick and Wolff hold that the specific sense of 'predominate' is that of 'serving as … the entry-point and the goal point of this strictly non-essentialist theoretical process' (2006: 29). The authors see social phenomena as constituting themselves in the process of mutual determination but deny any determination in the last instance (2006: 132). Following Althusser, they disregard the system's tendency towards its own supersession. For an assessment see Carchedi (forthcoming).

15 This presupposes the socialization of the means of production, both material and mental. But a formal, only juridical, socialization does not imply necessarily a different social content (as the experience of the Soviet Union teaches). I therefore disagree with the following: 'changes in one social factor – the presumed "most effective cause" – have been expected to usher in all manner of necessary effects which never materialized' (Resnick and Wolff, 2006).

16 In formal logic, results following from contradictory premises are called paradoxes. For example, 'John is a New Yorker, John says that all New Yorkers are liars'. If John tells the truth, all New Yorkers are liars, including John who however says the truth. Thus John tells both the truth and lies. This conclusion follows from contradictory premises. In fact, here we have an implicit third premise, that John tells the truth.

17 This holds also for induction. In formal logic, 'induction is rigorously impossible' (Lefebvre, 1982: 136).

18 And not only in the social sciences. Our daily routine is based on the assumption that certain premises will not change. If I plan to go to my work tomorrow, I do it on the assumption that I will not have been fired, that the means of transportation taking me there will work, that the sun will have risen, etc. To hold that we cannot assume that rate of profit will fall in the future on the basis that it has fallen in the past is equivalent to me staying home rather than going to my work simply because I cannot rigorously assume that I have not been fired.

19 Alcouffe holds that Marx liked mathematics as such because of its 'rigor and intellectual gymnastics' (1985: 41) and that the recreational, playful and philosophical aspects of mathematics were for him at least as important as his preoccupation with economics (1985: 40). Yanovskaja, the most important commentator of the *Manuscripts*, remarks that they offer no answer as to what moved Marx to move from the pursuit of algebra and commercial arithmetic to that of differential calculus (1969: 23).

20 Moreover, some writers question whether deduction plays such a role in mathematics as it is usually assumed. 'Formally axiomatized theories are the exception, not the rule, even in mature modern mathematics. Thus the deductivist picture of the mathematician deriving theorems from explicitly-stated axioms within a specified formal system is false.' (Larvor, 1999)

21 This point clashes with Alcouffe's opinion that a formal mathematical treatment of the law of the tendential fall in the profit rate would be 'particularly welcome' (1985: 37).

22 For a mathematically more precise formulation of Marx's method, see Marx (1983:195–6, note 7).

23 According to Lombardo Radice, Marx did not know the critical foundations of analysis, from Cauchy to Weierstrass, something which emphasizes his geniality in criticizing autonomously the 'mystical' foundations of calculus (1972: 274).

24 This view differs substantially from Alcouffe's opinion that 'the formalization of a social, and in particular of a critical science' should be sought in Hegel's *Science of Logic* (1985: 104).

25 In social reality, on the contrary, a social phenomenon can decrease in size until it becomes an individual phenomenon, a potential social phenomenon (see Part I, Section 2). But in social reality the notion of infinitesimally small is nonsensical.

26 In a letter to Marx dated 1882, Engels writes: 'the fundamental difference between your method and the old one is that you make x change into x', thus making them really vary, while the other way starts from x+h which is always only the sum of two magnitudes, but never a variation of a magnitude.'

27 It should be mentioned that the invention of calculus has its own social roots (determination). 'The invention of calculus, much as the birth of all modern science, followed closely on the birth of capitalism … mathematics was transformed from a science of constant quantities to the mathematics of varying quantities … The introduction of mechanical tools of production, from windmills and cranes to water pumps and machines to drill stones, the development of oceanic navigation, new military techniques, and the natural sciences in general demanded new knowledge-necessitating means of analysing and calculating motions (projectiles, free fall, planetary motion, accelerated motion, etc.)' (Gerdes, 1985: 17–18). Many examples of the social determination of natural sciences are given in Carchedi (1983: chapter 1; 2005). The pioneering work in this field is that of Hessen (1931).

28 For Zelený, exclusive focus on quantitative changes is a 'one-sided standpoint' (1980: 14) rather than being a specific class perspective.

29 This section is a revised version of Carchedi (2006: 370–75).

30 For the thesis that neo-classical economics shapes social and political organizations, see Finlayson et al. (2005).

31 All conceptions of equilibrium, from Walras' general equilibrium to the modern form of general equilibrium based on excess demand and supply, to the Marshallian partial equilibrium, are based on the same notion of economic rationality. Given that the focus of this section is not an assessment of these theories but the analysis of what they have in common, homo economicus and his class content, it will be sufficient to deal with some aspects of partial equilibrium theory. For general equilibrium, see Carchedi (1991: chapter 3).

32 Not only equilibrium but also the deviations from it (disequilibrium) are only powerful ideological notions with no scientific value. The terms 'non-equilibrium economics', as opposed to (dis)equilibrium, underlines this difference with the present approach.

33 Of course, markets do not clear. In this case the culprit is found in irrational behaviour, the paradigmatic case being the limits imposed by state intervention on economic man's freedom. See Paolucci (2001: 82).

34 To be credible, an ideology must not only relate to people's (to that group's) life experience but must also offer a plausible explanation of that life experience. It is through this plausibility that social groups can be made to accept a view of reality contrary to their own interests (rationality). The strength of an ideology vis-a-vis other views of reality is then measured by the degree to which it is generally accepted, i.e. by the degree to which it excludes other, alternative, views of reality. From this angle, neo-classical economics is a terribly strong ideology.

35 The argument that 'the critique is not founded upon the actual practice of policy makers or researchers' (Philp, 1999: 472) speaks volumes not only about the (ir)relevance of 'pure theory' but also about the infamies of the 'policy makers'.

36 For a critique showing, from a different perspective, the 'overall irrelevance of equilibrium economics' see Zafirovski (2002).

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